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Total No. of Questions : 07

M.Sc. Mathematics (Sem.-2)

ALGEBRA-II

Subject Code : MSM-201-18

M.Code : 75962

Date of Examination : 12-12-22

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C have THREE questions each.
3. Attempt any FOUR questions from SECTION - B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. Attempt the following :

a) Let R and S be two isomorphic rings. Show that $R[x]$ and $S[x]$ are also isomorphic.

b) Show that $2x + 1$ is a unit in $\mathbb{Z}_4[x]$.

c) Prove that the polynomial $f(x) = x^2 - 2x - 15$ is reducible over $\mathbb{Z}$.

d) Show that every field extension of prime degree is simple.

e) If a field F has q elements, then F is a splitting field of $x^q - x$ over its prime subfield.

SECTION-B

2. a) An integral domain R with unity is a UFD if and only if every non-zero, non-unit element is finite product of primes. (7)

b) Show that every ideal in $F[x]$, where F is a field, is a principal ideal. (without using the fact that $F[x]$ being a Euclidean domain is a PID.) (8)

3. a) Show that $x^4 + 1$ is not irreducible over $\mathbb{Z}_p$ for any prime p. (7)

b) Let F be a field and $p(x), f(x), g(x) \in F(x)$, where $p(x)$ is irreducible over F. Show that if $p(x) \mid f(x)\, g(x)$, then either $p(x) \mid f(x)$ or $p(x) \mid g(x)$. (8)

4. a) If L is an algebraic extension of K and K is an algebraic extention of F, then L is an algebraic extension of F. (7)

b) Prove that the ring $\mathbb{Z}$ of integers is a principal ideal domain. (8)

SECTION-C

5. a) Find the splitting field of $x^5 - 3x^3 + x^2 - 3$ over $\mathbb{Q}$. Also find the degree and the basis of it over $\mathbb{Q}$. (7)

b) Prove that every algebraic extension of a finite field is a separable extension. (8)

6. a) Find the Galois field of 9 elements. (7)

b) If F is a finite field and $m \in \mathbb{N}$, then there exist a field extension K of F such that $[K : F] = m$. (8)

7. a) Find the fixed field under $Aut(K)$, where $K = \mathbb{Z}_2$. (7)

b) Prove that any field extension K of F of degree two is a normal extension of F. (8)

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.